

## Thoughts on Grinold & Kahn's "Fundamental Law of Active Management"

By Max Darnell and Ken Ferguson

We have a natural interest in what Ronald Kahn has had to say about the implications of Grinold's "Fundamental Law of Active Management" for strategies such as TAA. Kahn suggests that,

*"Tactical asset allocation has a high skill hurdle...to generate an equivalent information ratio [equivalent to stock picking], the tactical asset allocator must demonstrate a higher level of skill."<sup>1</sup>*

We argue that Kahn's view is based on an interpretation of Grinold's law that (a) fails to evaluate strategies in a portfolio context and therefore draws the wrong conclusion, (b) ignores transactions costs, and, therefore, ignores the very thing that lowers the skill hurdle for TAA, and (c) overstates the difference in breadth that TAA has relative to stock picking. While our interest arises in response to the comments regarding TAA specifically, the issues we discuss have importance for all strategies, and the implications, we think, are important for both investment managers and plan sponsors alike.

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<sup>1</sup> Kahn, Ronald, "Seven Quantitative Insights into Active Management Part 3: The Fundamental Law of Active Management," BARRA Newsletter, Winter 1997.

### The Law

The most attractive aspect of Grinold "Fundamental Law of Active Management," is its simplicity. The productivity of an active manager will depend both on (1) his level of skill, and (2) how often that skill is put to use. Nothing could be more true than that.

The equation is simple enough. The active manager produces Information Ratio (IR), which is a measure of the value added per unit of risk. His skill in producing IR is defined by the Information Coefficient (IC)<sup>2</sup>, and the extent to which he applies his skill is termed Breadth, which is defined as the number of independent signals he derives.

$$IR = IC * \text{sqrt}(\text{Breadth})$$

Think of risk as the "input," IR as the productivity of a strategy, and value added as the output: at any given level of risk, the value added will be the IR times the risk. So, at any given level of risk, an active manager can either seek to improve his productivity by increasing the number of times he utilizes his skill by increasing the number of independent bets he makes (an increase in Breadth), or he can increase production by improving his skill (an improvement in IC).

The simplicity of Grinold's equation leads also to its greatest weaknesses. What is presumably a simplifying assumption, turns out to be a fundamental omission: the formula was derived in the absence of transaction costs, and when one takes transactions costs into account, both IC and breadth must be significantly redefined. Furthermore, the seemingly simplest term in the equation is actually the most complex: the concept of breadth depends upon a notion of

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<sup>2</sup> The Information Coefficient (IC) is defined as the correlation of ex ante forecasts with realized return.

independence that, as we will argue, must be taken into account, but measuring independence cannot be done with any real precision, and is thus subject to significant estimation error. Lastly, IC cannot be estimated a priori with any precision.

In short, the details that the equation hides turn out to be of paramount importance, and the implications of the equation for a strategy such as Tactical Asset Allocation turn out to be far more difficult to divine than a casual application of the equation might suggest. As it stands, the details tend to benefit a TAA strategy more than most other strategies.

Very importantly, the equation has typically been used in asking the wrong questions. Grinold's equation defines the expected IR of a single manager or strategy in isolation, and in doing so, it ignores essential portfolio considerations. The importance of skill and particularly breadth, even as Grinold defines them, lie in their potential to make a positive *marginal* contribution to fund Information Ratio. The equation, however, is oriented towards asking whether a manager, taken in isolation from the rest of a fund's active program, has an attractive IR. This is a mistake.

A positive, yet unspectacular IR which is uncorrelated with the rest of the portfolio can contribute *handsomely* to portfolio IR. It is even possible to have a materially *negative* IR which contributes positively to a portfolio IR. It is this marginal effect which should be used to establish the merits of different strategies as complements to a portfolio of strategies, not as an isolated component. Kahn himself makes this error in his comments about market timing strategies and Tactical Asset Allocation, despite the fact that Grinold and Kahn had themselves advocated that managers seek new sources of breadth. TAA's ability to increase the breadth and add to the marginal IR of a portfolio of strategies is one of its greatest strengths.

The simplicity of the formula invites two simple conclusions which are true, but misleading. If one strategy has more breadth (i.e., can make more independent decisions in any single year) than another strategy, then the former will have a better IR, assuming the two strategies have the same IC. And, if one strategy has a higher IC than another, the former will have a better IR, assuming the two strategies have the same breadth. Both statements are true, conditional on the assumption that one of the two variables is held constant. But, these assumptions are *big* assumptions.

The comments that follow will address four different issues pertaining to the Grinold equation.

- First, we'll comment on why IR needs to be considered in a portfolio context, particularly with respect to breadth.
- Second, we'll address the manner in which transactions costs change the definition of the terms in the equation, and the implications this can have for how the equation is considered.
- Thirdly, we'll discuss the nature and importance of independence as it relates to the concept of breadth. This is an area where Grinold and Kahn have already made some important observations, but there are some additional issues that merit our attention.
- And finally, we'll explore the flaws in applying Grinold and Kahn to the relative merits of TAA versus stock selection.

### Information Ratio and Breadth at the Margin

We need to ask whether breadth should be taken account of at the portfolio level or at the fund level. Kahn largely, albeit indirectly, answers this question himself when he states that,

"The law encourages managers to have an eclectic style. If a manager can find some independent source of information that will be of use, then he should exploit that information... At the same time, the sponsor

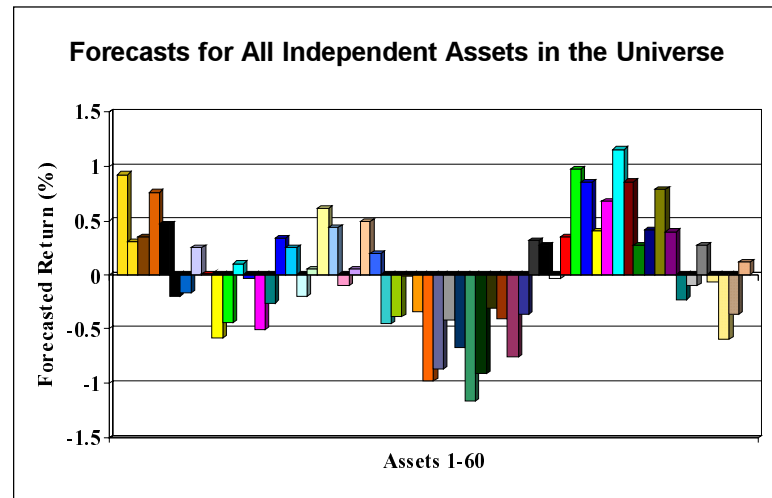
who hires a stable of managers has an incentive to diversify their styles in order to insure that their bets are independent. The way investment management is currently organized in the United States, the managers prepare the distinct ingredients and the sponsor makes the stew.” (pg. 129)

Kahn then later fails to recognize the “distinct ingredients” which TAA brings to the portfolio. Were a sponsor to identify a manager that has forecasting skill in an area where there is little breadth, but in an area that is also quite independent of the other active strategies employed by the fund, then the sponsor would be increasing the breadth of the fund’s active portfolio by including the manager, and thereby increasing the expected information ratio of the fund! From the perspective of the fund, risk should be allocated not according to the highest information ratios available, but according to the highest marginal impact on information ratios. Allocating a slice of the overall fund risk budget to yet another active manager whose style is only modestly different from the existing active managers simply because his IR is good does little to improve expected performance at the fund level.

Simple correlation can be a useful measure of how much breadth a strategy may bring to an existing portfolio of strategies. If a manager has a low observed IR, and even a low expected IR, there may be a role for the manager if the correlations with more conventional managers are low. As far as TAA is concerned, its correlation with other strategies tends to not only be low, but negative. When the conventional (e.g., average mutual fund) is outperforming sharply, the unconventional (e.g., TAA) can be very beneficial even if IR is neutral or slightly negative. Here, even a modest IR on a stand-alone basis can lead to a large improvement in **portfolio** IR, and the long-term IR for TAA has been substantial, not modest.

### Transactions Costs

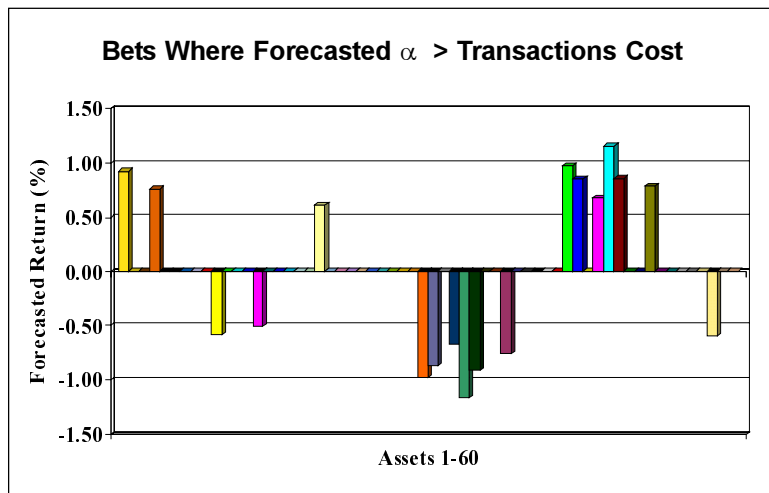
The conventional definition of an Information Coefficient (IC) is the measure of correlation of a forecast with its realized return. It is indicative of the manager’s skill in forecasting, but it is only partially related to the success a manager will achieve in managing a portfolio. This isn’t a mere subtlety. In most products (futures based TAA would



be an exception), transactions costs have a very significant role in determining how successful the manager turns out to be. They are not only a cost which offsets the profits obtained from trading on a successful forecast, but they also significantly reduce the universe of bets that the manager will take. These are issues that can, and very often do, turn a skillful forecaster into a failed asset manager.

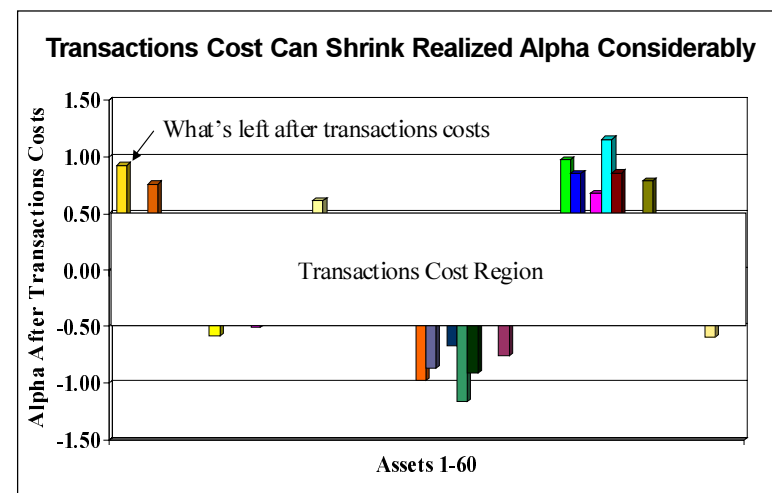
As an example, let’s say a manager forecasts returns to 60 stocks. We would measure the manager’s skill in forecasting – the IC – as the

correlation of all 60 forecasts with all 60 realized returns. According to the Fundamental Law of Active Management, if these forecasts all represented independent bets, then we would multiply the IC times the square root of 60 to derive the information ratio.



What this misses is the fact if transactions costs are non-trivial, the manager will not act on all 60 forecasts even if they are truly independent. For an active manager to take an active bet, the expected return must exceed the transactions cost paid for taking on the bet in the first place. Only a subset of the forecasts will be utilized, therefore. The greater the transactions costs, the fewer the number of independent bets there really are for the active manager. In the example below, if transactions costs are assumed to be 50 basis points, then only 18 of the 60 independent bets are really bets that the active manager may choose to employ. Even these 18 will have a sharply reduced payoff.

The ratio 18 out of 60 is, of course, entirely dependent upon the distribution of our randomly calculated forecasts, and our arbitrarily chosen transactions cost assumption. What will determine how much transactions costs will reduce the opportunity set is the ratio of the



forecasts' volatility to the transactions cost. Were forecasts normally distributed, with a standard deviation of 50 basis points, as in the example, and transactions costs of 50 basis points (i.e., were the standard deviation of returns equal to the transactions cost), then the reduction in independent bets from 60 to 16 or 17 would be quite expected. In short, all else equal, assets with minimal return volatility and high transactions costs will have very little effective breadth compared with assets having either very high return volatility or very low transactions costs.

This is not where the effect of transactions costs ends, however. The IC measures the relationship between the forecasted return and the realized return, but that is not what the active manager captures. The manager captures realized return *net of transactions costs*. This too changes dramatically the relationship between the IC and the IR wherever transactions costs are high relative to the variation in return.

Interestingly, this means that bets have a high serial correlation month to month, i.e., bets that will be held for many months, continuing to cumulate additional alpha from a single transaction, have a significant advantage over bets that change more frequently. When transactions costs are high, a measure of skill that takes account of a manager's ability to add value in excess of transactions costs may look quite different from the measure of skill implied by the IC. This appears to contradict a basic intuition of the Fundamental Law, in that there the IR is said to increase with the number of independent bets, whereas when transactions costs are significant, the realized gain in excess of transactions costs will grow if the same alpha can be achieved with *fewer* bets! (As a side note, managers who maximize IC without taking into account transactions costs are failing to maximize IR.)

Because of transactions costs, the alpha that is achieved with skilled forecasts can be much lower than what is implied by the conventional measure of IC. In order for Grinold's equation to be correct, we must first adjust the IC to reflect only those bets upon which the active manager may act upon, which is those bets where the forecasted return exceeds the transactions cost. This may or may not change the measure of IC depending upon whether the success of the managers forecasts varies directly with the magnitude of the forecasts. (We have seen very little evidence that quantitative models, for example, are any more or less reliable in the region of more extreme forecasts versus more normal forecasts, which would suggest that the IC will not obviously degrade after adjusting for this aspect of transactions costs.) The fact that the number of independent bets that the manager would actually

act upon falls with transactions costs implies that the measure of breadth must fall. Breadth is properly defined as this smaller universe of active bets, not the larger universe of active bets that the manager has forecasts for. Finally, the IC must reflect the correlation of forecasted alpha net of transactions costs with realized alpha.

The significance of lower transactions costs just can't be ignored. Clearly strategies that face lower transactions costs suffer far less from these issues than strategies where transactions costs are high. Because we trade so cheaply when trading futures and forwards, our conventional measure of IC is not eroded anywhere near as quickly as it is for an investor who trades in the cash markets, whether they are trading stocks, industry sectors, or equity indices, so in this sense, the IC hurdle is lower for a TAA manager than it is for a conventional stock manager when all else is equal.

### The Independence of Signals

Grinold & Kahn describe the number of bets as "breadth," and they define breadth as "the number of independent forecasts of exceptional return [made] per year."<sup>3</sup> Breadth, in active management, is a slippery concept, however, and Grinold and Kahn do seek to address this issue, in part, in their paper. Independence, as Grinold and Kahn themselves point out, is not as simple as the number of assets a manager has at his disposal, nor is it simply the number of active positions. These can both relate to breadth, but only to the extent to which they are independent of one another. What does independence mean? The assets or active positions must themselves be independent, and the factors that drive the active decisions must also be independent if they are to contribute to the breadth of the strategy.

<sup>3</sup> Richard C. Grinold and Ronald N. Kahn, *Active Portfolio Management*. Irwin Professional Publishing, 1995, pp. 117-135.

The correlations of market indices are lower than the correlations of stocks within an index. As a result, an active position on the average individual equity index will contribute more to breadth than the average stock within an equity portfolio will contribute. In short, strategies that focus on assets with lower correlation have more potential breadth associated with each individual asset than strategies focused on highly correlated assets. Tactical Asset Allocation portfolios are advantaged by this in that not only are equity market indices less correlated with each other than the individual stocks within those indices are to each other, but also because Tactical Asset Allocation is able to take advantage of the lower correlations across asset classes by taking active positions on stocks, bonds and currencies simultaneously.

Breadth also depends on independence in the factors that drive the active decisions. A single factor can, and most commonly does, drive several different active positions. Common region, country, sector, industry, style or economic factors will often drive multiple active positions, and when this is the case, there is far less independence than is implied by the number of assets or active positions involved. The truth about most active equity processes is that there are really not very many factors that drive the decisions being made about the many stocks in a portfolio. So the number of active stock decisions, for example, will not be very indicative of the number of independent bets.

Even in fundamental, bottom-up strategies where individual analysts may track individual companies, one will find that there are a great many common factors driving their separate analysis. Economic factors such as interest rates, energy prices, exchange rates, inflation, labor costs and GDP growth will frequently play a common role in their decisions, as will information about sectors or industries such as new production technologies, the price of raw inputs, changes in industry or sector production capacity, and etc. Consider the case where individual stock analysts working separately all upgrade their views on GM, Chrysler and Ford because they have observed in common

the fact that the health of the economy is improving and the fact that expectations for car sales have improved. Because their recommendations on GM, Chrysler and Ford are all being driven by the same information, these seemingly independent factors are not, in fact, independent.

The independence of the assets themselves can be estimated as can the independence of the information that underlies a manager's forecasts. Correlation analysis can tell us a great deal about whether the assets, for example, have behaved more or less independently in the past or not. Unfortunately, correlations are usually subject to significant variation. They tend to hold in some market environments but break down in others, and there is a tendency for correlations to shift significantly during the most extreme, and most important market events. The apparent independence of information driving our forecasts or the assets themselves can easily be far different in truth than they appear to be when we measure their correlation over the very recent path. The point is that independence, and therefore breadth, is not the simple, easy to use term that it might appear to be, and Grinold's equation, while still useful conceptually, will not be very reliable in application.

As an aside, one should be careful not to confuse assets held in a manager's portfolio with active assets. In a typical equity portfolio, something on the order of two-thirds or more of the assets held in the portfolio (as measured by capitalization weight, and depending on the targeted tracking error level) will be held not as active bets – even though they may be deviant in magnitude from benchmark weight – but rather they will be held instead in the effort to manage to a tracking error constraint. From the very start, this means that the number of seemingly active positions tend to be far fewer than the number of assets in the portfolio, and the number of *independent* active positions will be even smaller when one considers the correlations of the assets and the commonality of the decision drivers.

With regard to Tactical Asset Allocation specifically, it bears mentioning that Kahn's recommendation against TAA is based on an entirely different measure of breadth than what is appropriate for what we, at First Quadrant, tend to mean by TAA. Only in the US is the term "TAA" associated with strategies that seek to add value through managing exposures to three domestic asset classes, US stocks, US bonds and cash. The three decision points means that there is unlikely to be much breadth in a TAA program, and since TAA managers tend to hold their bets in place over multiple months, the frequency of independent bets is also low.

In a global context, however, "TAA" refers to strategies that do much more than simply seek to manage exposure to three domestic asset classes. Asset class selection is only one aspect of what TAA managers do. The product also seeks to add value through the selection of markets within the asset classes, choosing amongst both equity and fixed income market exposures, while also managing the currency exposures across this same set of markets. In the universe of developed markets, there are 22 equity markets to choose from. Amongst those, there are twelve markets that can be considered to have highly liquid, exchange traded futures associated with them. Add to that seven highly liquid exchange traded fixed income futures contracts and eleven liquid currency forwards contracts and you have ten times the number of assets in the universe than you have in the domestic TAA product that Kahn's comments were directed at. Clearly, TAA as we most typically employ it has far more breadth than the TAA that Kahn was concerned with.

### Implications for Tactical Asset Allocation

Grinold and Kahn suggest that TAA cannot have an attractive IR, unlike stock selection, because the "breadth" of the TAA decision is so much smaller than the breadth of the stock selection decision. If one strategy has more breadth (i.e., can make more independent

decisions in any single year) than another strategy, then the former will have a better IR, *assuming the two strategies have the same IC*. If we're right 55% of the time on two decisions a year, we'll have a much lower IR than if we're right 55% of the time on 1000 decisions a year. This is true, conditional on the assumption that IC is held constant. But, this is flawed assumption.

Few investors would disagree with the notion that, the smaller and narrower the decision, the more "noise" we'll find in the data. More noise means a weaker signal/noise ratio, even for a highly effective strategy. A weaker signal/noise ratio means a lower IC. This means that breadth and IC are interrelated. A strategy with enormous breadth (i.e., many small, short-horizon decisions) *will* have a small IC; a strategy with low breadth (i.e., the big, long-horizon decisions) *may* have a large IC. So, greater breadth does *not* assure a higher IR, because the ICs likely *will* be very, very different.

A strategy which has a very short investment horizon, covering individual assets (e.g., will Microsoft rise or fall tomorrow?), *cannot* have a large IC, especially net of trading costs. A strategy which has a long investment horizon, covering a broad category (e.g., will stocks beat bonds over the next decade by a larger or smaller margin than the last decade?) *can* have a large IC, if the analytic framework we use is sound. This is an important Achilles' heel in the Grinold/Kahn argument against TAA. TAA makes fewer decisions in a year, acting on three asset classes, than stock selection, acting on 1000 assets. But, this means that the TAA IR will be lower than the stock selection alpha *only* if we expect the ICs to be comparable. The conclusion is flawed because the supposition of equivalent ICs is not reasonable. And, globalizing the TAA decision can restore breadth to the process.

As to TAA specifically, what TAA brings to the equation is not only a positive historical IR, much lower transactions costs (when implementation is derivatives-based), and, and a unique breadth. Lower

transactions costs mean that there will exist a tight relationship between skill and results. Strategies that trade at very low transactions cost have a large advantage in that they are able to (a) capture a larger share of their potential alpha because they don't have to disregard forecasts for being too expensive to exploit, and because they are able to (b) keep more of their alpha after transactions costs are paid for the opportunities that are exploited. The IC "hurdle" is lowered for these strategies as a result, a conclusion which directly opposes the conclusion Kahn drew. It is interesting to note that most TAA managers have added value in the last fifteen years, while most equity managers have not, despite the ostensible breadth advantage.

As to the matter of breadth, the breadth that TAA offers may, in fact, be an uncomfortable breadth, but the asset management activity is not one which we should expect will reward comfort. It is the uncomfortable decisions that should be most profitable. Furthermore, when strategies are uncomfortable, they are most likely to be uncorrelated and therefore most beneficial on a portfolio IR basis because of the breadth that they carry.

Part of the discomfort with TAA derives from the fact that it doesn't pay off as often as some strategies might that focus on those assets that don't have much impact at the fund level. A 1% IC on daily stock prices can suffice for profitable trading (if trading costs are low enough), while a 10% IC on TAA might take 10 years to show significant enough results to merit confidence. Conversely, a 10% IC on TAA might be far easier to achieve than a 1% IC on daily stock prices. Our own record for TAA would suggest a much higher IC on the GTAA decisions than 10%. Strategies that don't pay off very often will be uncomfortable, and there will be a natural tendency of managers to choose to avoid such strategies.

## Conclusion

The insights that Grinold and Kahn have offered are important and useful, but care must be taken to account for the very important implications that both independence and transactions costs have on breadth, and that transactions costs have on the cost adjusted measure of skill as represented by the unconventional measure of transactions cost adjusted IC.

Care must also be taken not to overrely on a metric which suggests precision in a very imprecise world. We should not be at all surprised to observe strategies which have little apparent breadth having very significant IR's, and we should not be surprised by strategies with significant breadth having poor IR's. Skill will surprise or disappoint us, of course, but so will our estimation of a manager's breadth as we find his breadth to be more or less than we will have guessed. Neither future breadth, nor future IR, can be gauged with any precision whatsoever. What's a manager's likely future IR? Is past IR a good predictor? No. Are subjective evaluations of a manager's discipline? Yes, a bit. Is it helpful to consider context, with an evaluation of whether manager is doing well when he would be **expected** to do well, or badly when s/he would be **expected** to do badly? Yes, a bit. How much time is needed to persuasively detect skill in any strategy? Longer than any client will have patience to wait.

Most importantly, the equation should be used to ask the question, "What is the *marginal* impact this additional manager or this additional strategy will have on fund performance?", rather than asking, "What is the IR of this strategy or manager when measured in isolation?"

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